Gradient Decent Based Polynomial Regression

Loic Guegan

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First, choose a polynomial function $h_w(x)$ according to the data complexity. In our case, we have:

$$h_w(x) = w_1 + w_2 x + w_3 x^2 \tag{1}$$

Then, we should define a cost function. A common approach is to use the **Mean Square Error** cost function:

$$J(w) = \frac{1}{2n} \sum_{i=0}^{n} (h_w(x^{(i)}) - \hat{y}^{(i)})^2$$
(2)

Note that in Equation 2 we average by 2n and not n. This is because it get simplify while doing the partial derivatives as we will see below. This is a pure cosmetic approach which do not impact the gradient decent (see here for more informations). The next step is to $min_w J(w)$ for each weight w_i (performing the gradient decent). Thus we compute each partial derivatives:

$$\frac{\partial J(w)}{\partial w_1} = \frac{\partial J(w)}{\partial h_w(x)} \frac{\partial h_w(x)}{\partial w_1}$$
$$= \frac{1}{n} \sum_{i=0}^n (h_w(x^{(i)}) - \hat{y}^{(i)})$$
(3)

similarly:

$$\frac{\partial J(w)}{\partial w_2} = \frac{1}{n} \sum_{i=0}^n x(h_w(x^{(i)}) - \hat{y}^{(i)}) \tag{4}$$

$$\frac{\partial J(w)}{\partial w_3} = \frac{1}{n} \sum_{i=0}^n x^2 (h_w(x^{(i)}) - \hat{y}^{(i)})$$
(5)